DECOMPOSING THE GAINS FROM TRADE IN THE PRESENCE OF TIME-CONSUMING CONSUMPTION

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We examine the decomposition of the gains from trade when consumption is time consuming in a simple open economy setting. While trade remains welfare improving, the sources of trade gainfulness differ from those in conventional trade models. In particular, the conventionally defined exchange (consumption) and specialisation (production) gains vanish. There are, however, positive gains from time reallocation (away from production toward consumption) and specialisation associated with this time reallocation.

Keywords: Gossen, Becker, Consumption time constraint, Gains from trade, Time reallocation

JEL Classification: F10, F11, D11

1. Introduction and Context

In a previously published article in this journal (Tran-Nam, 2011) it has been argued that the theory of time allocation dates back at least to Gossen, a Prussian civil servant, whose lifetime work was contained in a single book published in 1854. This book was a treatise on economic laws (i.e., laws dictated by nature) and moral laws (i.e., rules of human conduct). In his work, Gossen (1854; 1983) saw that what is ultimately scarce is time alone. In his view, even in the land of Cockaigne where commodities are freely available in unlimited quantities, there will still be an economising problem.

Gossen’s book was rather hastily completed, very difficult to read and also a commercial failure (Georgescu-Roegen, 1983). As a result, his ideas were little known both within and outside Germany. When his work was brought to the attention of fathers of the Marginal Revolution, Jevons (1879: pp. xxxv–xlvi) and Walras (1885) praised Gossen’s contribution to the theory of marginal utility but they did not pay attention to his primary emphasis on the constraint of time. For a long time, Gossen’s contributions to economic theory were largely ignored by mainstream economists.

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Much later, Becker (1965) proposed a general theory of time allocation in which households act as productive agents who combine time and market goods to generate vectors of basic commodities. In introducing the concept of a household production function, Becker generalised the Gossenian consumption time constraint. Since Gossen’s book was obscure to English-speaking economists until the early 1980s, it is not surprising that Becker did not refer to the work of Gossen. Sadly, this neglect has surprisingly persisted until today, despite the availability of Blitz’ English translation of Gossen’s book in 1983 with a very comprehensive introduction by Georgescu-Roegen (1983).

Gossen’s consumption time constraint has been mainly discussed in the context of closed economies; see, for example, Georgescu–Roegen (1983), Niehans (1990) and Steedman (2002). In the case of open economies, Kemp (2009) demonstrated that the normative theory of trade, including the well-known Grandmont–McFadden (1972) and Kemp–Wan (1972) propositions, survive the incorporation of a Gossenian time constraint. More recently, Tran-Nam (2012) investigated the pattern of and gains from trade in a Ricardian model incorporating a consumption time constraint.

While trade remains potentially gainful, little is known about the source of gains from trade in the presence of time-consuming consumption. In particular, do the exchange (consumption) and specialisation (production) gains remain valid when consumption is itself time-consuming? While this question has been partly considered in the small-country case of a Ricardian world (Tran-Nam, 2012), it is unclear whether the findings can be extended beyond the one-factor, linear production technologies case.

The principal aim of this paper is to decompose the gains from trade in a Heckscher–Ohlin–Samuelson (HOS) model under the assumption that the act of consumption itself takes time. To focus on the decomposition of trade gains and for ease of graphical illustration, the analysis is based on a simple model of two countries, two goods and two factors with representative agents. However, it will soon be apparent the basic reasoning of the analysis in this simple world would continue to hold in the more complex cases with higher dimensions or certain types of heterogenous agents.

The problem under study is not only theoretically interesting but also empirically relevant. Specifically, there has been growing anecdotal evidence that workers, especially those in developed nations, feel increasingly stressed because they are time poor (see, for example, Schulte, 2014). Workers have been spending more time at work so that they do not have sufficient time for leisure and consumption. This suggests that it is no longer sensible to dismiss the importance of balancing between working and consumption.

The remainder of this paper is organised as follows. In Section I we consider how the consumption time constraint can be interpreted and formulated mathematically. Section 2 examines a simple closed economy with two goods, two factors, representative agents and time-consuming consumption. A simple HOS model is then analysed in Section 3. It is shown that the exchange (consumption) and specialisation (production) gains, as conventionally defined in the literature, both vanish. There are, however, a time allocation gain arising from reallocating more labour time to consumption, and a specialisation gain associated with this reallocation. Section 4 concludes.
2. The Consumption Time Constraint

Apart from the literature associated with the work of Gossen and Becker (1960), remaining economic models, including the labour–leisure choice model or theory of trade with variable labour supply (see, for example, Kemp and Jones, 1962; Martin and Neary, 1980, Woodland, 1982; Mayer, 1991), all fail to recognise that consumption takes time. Even when the importance of a consumption time constraint is recognised, there is still a debate about how time should be treated. Is it an input or merely a context? Several authors (see, for example, Winston, 1982: 164; Steedman, 2001: 5) have argued that time is a context in Gossen case while it is an input in Becker case. In this respect, the approaches of Gossen and Becker are said to be fundamentally different. But regardless of how one may interpret the role of time in consumption, it can be shown below that both approaches are similar, at least from a mathematical formulation point of view.

The fact that consumption takes time adds another constraint to the consumer choice problem. Since Gossen did not express his idea in mathematical form, it is somewhat unclear how his consumption time constraint should be formulated. A seemingly plausible way is to start with \( L_i^c = g_i(X_i) \) and \( \sum_{i=1}^{n} L_i^c \leq L^c \) where \( X_i \) refers to the amount of the \( i \)-th good purchased, \( L_i^c \) to the number of time units required to consume \( X_i \), \( L^c \) is the total number of time units available for consumption, \( g_i(0) = 0 \) and \( g_i(0) \) is strictly increasing \((i = 1, 2, \ldots, n)\). Thus, the simplest formulation of the Gossen consumption constraint, as adopted by Niehans (1995) and Steedman (2002), is that \( \sum_{i=1}^{n} a_i X_i \leq L^c \) where \( a_i \) stands for the number of time units required to consume one unit of the \( i \)-th good \((i = 1, 2, \ldots, n)\).

Expressed in this way, it is not difficult to see that the Gossenian approach can be viewed as a special case of Becker’s production function approach, \( C_i = h_i(X_i, L_i^c) \) and \( \sum_{i=1}^{n} L_i^c \leq L^c \), where \( C_i \) refers to the amount of the \( i \)-th ‘basic commodity’ \((i = 1, 2, \ldots, n)\). More specifically, if all \( h_i \) take the simple Leontief form, i.e., \( C_i = \min \{X_i, L_i^c/a_i\} \) \((i = 1, 2, \ldots, n)\), then the Beckerian model simplifies into the Gossenian model.

The approach adopted in this paper is both Gossenian and Beckerian. It is Gossenian in the sense that all consumption technologies take the simple Leontief form. But it is also Beckerian in the sense that time is formally treated as a labour input to be expended in the consumption process.

3. The Closed Economy

We begin by describing the closed economy and deriving its autarkic equilibrium. The economy is populated by identical agents who are endowed with labour and capital where labour (measured in time units) can be alternatively allocated between production and consumption\(^1\) while capital is employed in production only. For each agent, consuming and working cannot be undertaken simultaneously. Aggregate endowments of labour and capital are denoted by \( \bar{L} \) and \( \bar{K} \) respectively.

\(^1\) The act of consumption can be broadly interpreted to include search, purchase, preparation and consumption.
In the output market, price-taking, competitive firms produce two private goods with the aid of two essential inputs, labour and capital. The aggregate production functions are written as \( Q_i = F_i(L_i, K_i) \) where \( Q_i \) is the output of the \( i \)-th sector, and \( L_i \) and \( K_i \) are the amounts of labour and capital employed in sector \( i \) respectively (\( i = 1, 2 \)). It is assumed that \( F_i(0, 0) = 0 \) and \( F_i \) is a twice differentiable function that exhibits constant returns to scale, positive, diminishing marginal products, and strictly decreasing marginal rate of technical substitution along any isoquant. Thus, \( F_i \) is concave. The problem facing the producer of good \( i \) is to maximise \( P_i Q_i - W L_i - R K_i \) by the choice of \( (L_i, K_i) \) subject to the production constraint and the non-negativity of inputs where \( P_i \) is the price of good \( i \) (\( i = 1, 2 \)), and \( W \) and \( R \) are the wage and rental rates respectively.

Let \( X_i \) denote the quantity of the consumption good \( i \) that the consumer purchases, and let \( C_i \) denote the quantity of the ‘final’ consumption good \( i \) that the consumer wishes to enjoy. To transform \( X_i \) into \( C_i \), the consumer needs to use another input, called ‘consumption time.’ The Leontief consumption technology is assumed, i.e., \( C_i = \min\{X_i, L_i^c/a_i\} \) where \( L_i^c \) refers to the number of time units required to consume \( X_i a_i > 0 \) is the technological coefficient associated with good \( i (i = 1, 2) \). For example, suppose \( a_1 = 3 \), then if the consumer wants \( C_1 = 1 \), he/she needs to choose \( X_1 = 1 \) and \( L_1^c = 3 \). Note that information search, time spent on purchasing, etc, can be incorporated into the technological coefficients of this Leontief conversion technology.

Since agents are identical it is possible to speak of a social utility function. The utility function is summarised as \( U(C_1, C_2) \) where \( U \) is supposed to be homothetic and twice differentiable with positive marginal utility and strictly diminishing marginal rate of substitution along any indifference curve. Thus, \( U \) is strictly quasi-concave. It is also assumed that \( \text{MRS}_{21} \equiv -dC_2/dC_1 \rightarrow \infty \) (\( 0 \)) as \( C_1/C_2 \rightarrow 0 (\infty) \).

The consumer’s problem is to maximise \( U(C_1, C_2) \) by the choice of \( (C_1, C_2) \) subject to the time constraint \( \sum_{i=1}^{n} L_i^c \leq L^c \) and financial constraint where the financial constraint varies depending on whether the economy is closed or open. Two remarks deserve mention. First, while pooled consumption (eating or watching TV together) is allowed for, consuming jointly does not give an agent more satisfaction than consuming alone. Secondly, a vast majority of leisure-related activities, such as reading a book, or skiing, involves combining intermediate goods with scarce consumption time. Such active leisure as well as passive leisure (where neither capital nor a commodity is required) can be straightforwardly accommodated in the present model.

In this simple economy, the competitive equilibrium can be obtained as the solution to the central planner’s problem. In the absence of international trade, focusing on the real side of the economy, the model constraints can be expressed in the form of inequalities as follows:

\[
\begin{align*}
F_i(L_i, K_i) - Q_i &\geq 0, \quad i = 1, 2 \\
Q_i - X_i &\geq 0, \quad i = 1, 2 \\
\min\{X_i, L_i^c/a_i\} - C_i &\geq 0, \quad i = 1, 2 \\
\bar{L} - L_1 - L_2 - L_1^c - L_2^c &\geq 0
\end{align*}
\]
\[ K - K_1 - K_2 \geq 0 \]  
\[ K_i \geq 0, L_i \geq 0, L'_i \geq 0, Q_i \geq 0, X_i \geq 0, C_i \geq 0, \quad i = 1, 2. \]

where (2) and (3) can be thought of as the financial and consumption time constraints respectively.

Any vector \( y = \{C_1, C_2, X_1, X_2, Q_1, Q_2, K_1, K_2, L_1, L_2, L'_1, L'_2\} \) that satisfies (1)−(6) is said to be feasible. We define the set \( S \) of feasible allocations as

\[ S \equiv \{ y \in \mathbb{R}^{12}_+ \text{ such that (1) to (6) hold} \} \tag{7} \]

By a theorem in Takayama (1974) on concave programming, if all inequality constraints are expressed in the form \( z_j(...) \geq 0, j = 1, 2, \ldots, m \), and if each of these \( z_j \) is a concave function (possibly linear), then the feasible set is a convex and non-empty set.

Next, we define the projection of the feasible set \( S \) into the feasible final-goods space \((C_1, C_2)\). Call this set \( S_C \).

\[ S_C \equiv \{(C_1, C_2) \in \mathbb{R}^2_+ \text{ such that } y \in S\} \tag{8} \]

It is well-known that the projection of a convex set in \( \mathbb{R}^m_+ \) into \( \mathbb{R}^2_+ \) is itself a convex set. \( S_C \) is therefore also a non-empty and convex set. The maximisation of a strictly quasi-concave function \( U(C_1, C_2) \) over a convex set \( S_C \) yields a unique solution \( (C_1^*, C_2^*) \) (Takayama, 1974). This result can be summarised as

**Proposition 1.** There exists one and only one autarkic equilibrium in this economy.

The upper boundary of \( S_C \) can be expressed as \( C_2 = \phi(C_1) \) where \( \phi \) is concave and strictly decreasing. The function \( \phi \) captures information about resource endowments, and production and consumption technologies. The graph of \( \phi \) is the locus of all maximal consumption points of the economy under autarky and can thus be thought of as the autarkic consumption possibility frontier (CPF).

Let \( L_M \equiv \bar{L} - L'^C \) be the amount of labour time devoted to manufacturing. We define the production possibility set

\[ S_Q \equiv \{(Q_1, Q_2) \in \mathbb{R}^2_+ : Q_i \leq F_i(L_i, K_i), \quad L_1 + L_2 \leq L_M \text{ and } K_1 + K_2 \leq \bar{K}\}. \tag{9} \]

The upper boundary of the set is the production possibility frontier, and can be represented by the strictly decreasing and concave function \( Q_2 = \psi(Q_1; L_M, \bar{K}) \).

We are particularly interested in the specific curve \( Q_2 = \psi(Q_1; L'^*_M, \bar{K}) \), where \( L'^*_M \equiv \bar{L} - L'^* \) and \( L'^* \equiv L'_1^* + L'_2^* = \sum_{i=1}^2 a_i X_i^* = \sum_{i=1}^2 a_i C_i^* \) refers to the equilibrium time devoted to consumption. This curve generates the relevant relative supply curve \( Q_2/Q_1 \), as an increasing function of their relative price \( p \equiv P_2/P_1 \). Since we know \( X_2^*/X_1^* = C_2^*/C_1^* \), we can pin down the equilibrium relative supply, \( Q_2^*/Q_1^* = X_2^*/X_1^* \), under autarky. This point

\[ \text{It is well-known that the function } \min\{\ldots\} \text{ is a concave function.} \]
on the relative supply curve determines the equilibrium relative price $p^* = P^*_2/P^*_1$. Without loss of generality, let $P^*_1 = 1$ so that $p^* = P^*_2$. Given the relative price $p^*$, we can now work backward to find the equilibrium factor prices in terms of good one, $W^*$ and $R^*$, by using the conditions that the price of each good is equal to its unit cost $c_1(W, R) = 1$ and $c_2(W, R) = p^*$.

The autarkic equilibrium can be illustrated graphically by noting that, for given any value $L_M \in (0, L)$, the PPF, $Q_2 = \psi(Q_1; L_M, K)$, and the consumption-time budget line, $Q_2 = [L - L_M - a_1Q_1]a_2$, can be plotted. These two curves may not intersect at all or intersect (at one or two points) or tangential in the positive quadrant. The intersection and tangential cases are illustrated in Figures 1 and 2, respectively, where the curve $P^*P$ stands for the PPF and the line $T'T$ for the consumption-time budget.

When $P^*P$ lies everywhere below (above) $T'T$, agents devote too little (much) time to working so that after consuming all income they still have some surplus time (or they do not have sufficient time to consume all income). This can be referred to as an income-poor (time-poor) situation. Neither an income-poor nor a time-poor allocation can be optimal because economic agents can always consume more of both goods by devoting more (less) time to production. Thus, only points of intersection ($K$ in Figure 1-a or $K'$ and $K$ in Figure 1-b) or tangency ($K$ in Figure 2) can be maximal consumption points under autarky. As $L_M$ varies from $0$ to $L$, the locus of such points traces out the CPF mentioned previously.
4. The Trading World

Now we turn our attention to international trade. For simplicity, the world is supposed to consist of two countries, called home (H) and foreign (F). Each country produces two identical private goods with the aid of two essential inputs, labour and capital, as described in Section 2. It is assumed that labour, capital and final consumption goods cannot be traded, but trade in produced goods is free, costless and balanced.

Suppose that the autarkic equilibrium relative price of produced goods in H, $p_H^*$, differs from that in F, $p_F^*$. Then there is an incentive for the two countries to trade. Let the world terms of trade (relative price of intermediate two in terms of intermediate good one) be denoted by $p^w$. The financial constraint (2) in the closed economy case of H now becomes the balance of trade constraint:

$$(Q_{1H} - X_{1H}) + p^w(Q_{2H} - X_{2H}) \geq 0$$

Given $p^w$, it is possible to determine the excess demand of each output. This can be easily done if H’s post-trade production is not completely specialised. When both goods are produced, the price-equals-cost conditions uniquely determine the factor prices $W_H$ and $R_H$: $W_H = W_H(p^w)$ and $R_H = R_H(p^w)$. Then the virtual national income of H, which includes the imputed value of labour used in consumption activities, is $Y_H(p^w, \bar{L}_H, \bar{K}_H) = W_H(p^w)\bar{L}_H + R_H(p^w)\bar{K}_H$. The prices of the two final consumption goods are the world prices plus opportunity costs of consumption, i.e., $p_1H = 1 + a_{1H}W_H(p^w)$ and $p_2H = p^w + a_{2H}W_H(p^w)$.

It is well known in the trade literature that the compensating-variational measure of gains from trade can be decomposed into consumption (exchange) gain and production (specialisation) gain (see, for example, Bhagwati and Srinivasan, 1983: 167–168). Exchange gain refers to the gain that a trading country can enjoy when, in free trade, it was constrained to produce at the autarkic production bundle whereas consumption was allowed to be at international prices. Specialisation gain then refers to the additional gain a trading country can enjoy from being allowed to shift production under free trade from the autarkic equilibrium.
to the post-trade equilibrium according to the principle of comparative advantage.

In the present model, exchange gain can be interpreted as the gain that would accrue if the home country continued to allocate $L_{MH}^*$ to production and produce the autarkic equilibrium bundle $(X_{1H}^*, X_{2H}^*)$. Under free trade, the home country can now afford a bundle which lies beyond its autarkic equilibrium PPF and belongs to an indifference curve that is higher than the autarkic equilibrium one. However, holding the amount of time allocated to consumption constant at $L_{CH}^*$, agents in the home country are unable to have sufficient time to fully consume that new bundle. This is depicted in Figure 3 where any bundle along the dotted ray $A_hB$ or $A_hC$ (representing world terms of trade) is financially affordable but not time feasible to the home country. Thus, there is no exchange gain in the present model. In fact, more generally, there is no exchange gain for any given allocation of time to consumption.

**Figure 3:** Neither exchange gain nor (conventional) specialisation gain from trade.

In general, the home country would become time poor under trade if the autarkic level of time allocation between consumption and production was maintained so that neither exchange gain nor specialisation gain would be possible. Thus, for trade to be gainful, there must be first a reallocation of labour time away from production toward consumption. This would eliminate, or at least lessen, time poverty and then allow a reallocation of inputs between the two productive sectors according to the principle of comparative advantage. In this sense, there is a specialisation gain associated with time reallocation between consumption and production.

If a trading country devotes less time to production, its PPF shifts downward relative to the autarkic PPF. By producing at a point where the marginal rate of transformation is equal to the world terms of trade and trading at international prices, the home country can financially afford a bundle that lies beyond its autarkic equilibrium PPF. At the same time, having more time available for consumption, economic agents in the home country now also have sufficient time to fully consume this bundle. Note that the post-trade production point
is consistent with the theory of comparative advantage in the sense that if \( p_H^* > (\leq) p_w \), the home country will export good two (one).

The above argument is illustrated in Figure 4. In this Figure, H is assumed to be relatively more labour-abundant than F. H’s endowments of capital and labour time are \( K_H = 200 \) and \( L_H = 400 \) respectively whereas for F, \( K_F = 320 \) and \( L_H = 300 \). For simplicity, we also assume that \( a_1 = a_2 = 1 \) for both H and F. Further, good one is assumed to be always more labour intensive than good two.

![Figure 4: Time allocation and specialisation gains from trade \((a_1 = a_2 = 1)\)](image)

Autarky: \( L_{MH}^A = 220; L_{CH}^A = 180; L_{MF}^A = 120; L_{CF}^A = 180 \)

Post-trade: \( L_{MH}^T = 200; L_{CH}^T = 200; L_{MF}^T = 100; L_{CF}^T = 200 \)

H’s autarkic equilibrium is at point \( A_H \) where 120 units of good one and 60 units of good two are produced. Thus 180 units of labour will be needed in consuming these outputs. H’s labour employment in manufacturing under autarky is \( L_{MH}^A = 400 – 180 = 220 \). Its PPF is \( Q_2H = \psi(Q_{1H}, L_{MH}^A, K_H) = \psi(Q_{1H}; 220, 200) \). F’s autarkic equilibrium is at point \( A_F \) where 60 units of good one and 120 units of good two are produced. Thus 180 units of labour will also be needed in consuming these outputs. F’s labour employment in manufacturing under autarky is \( L_{MF}^A = 300 – 180 = 120 \). Its PPF is \( Q_2F = \psi(Q_{1F}, L_{MF}^A, K_F) = \psi(Q_{1F}; 120, 320) \).

As drawn, H’s autarkic welfare is the same as that of F (note that both countries have the same autarkic consumption-time budget line).
Now allow both countries to trade. H will export good one (which it has a comparative advantage) and import good two, and F will export good two and import good one. Both countries become wealthier and can attain the higher level of welfare as depicted by point $B$ in Figure 4. At $B$ each country consumes 100 units of good one and 100 units of good two. This means each of them needs 200 units of labour time for post-trade consumption. This also implies that, as a result of trade, less labour is allocated to manufacturing in both countries: $L_{MH}^T = 400 - 200 = 200$ and $L_{MF}^T = 300 - 200 = 100$. Their PPFs are shifted downward (not drawn) to $Q_{2H} = \psi(Q_{1H}; L_{MH}^T, \bar{K}_H) = \psi(Q_{1H}; 200, 200)$ and $Q_{2F} = \psi(Q_{1F}; L_{MF}^T, \bar{K}_F) = \psi(Q_{1F}; 100, 320)$. H and F post-trade production points are $X(170, 30)$ and $Y(30, 170)$ respectively.

In summary, we may now state

**Proposition 2.** There is neither exchange gain nor (conventional) specialisation gain in this simple trading world in which consumption takes time. However, there is a time allocation gain (under which a trading nation will devote more time to consumption relative to autarky) and a specialisation gain associated with this time reallocation.

### 4. Concluding Remarks

The present paper has examined the gains from trade in a simple HOS world when consumption takes time. It began by considering the equilibrium of a closed economy with representative agents, two goods, two factors and Leontief consumption technologies. It was shown that key results of the conventional model would carry over to the present model with the traditional PPF being replaced by the CPF which can be defined as the locus of all maximal consumption points under autarky.

Turning to international trade, the introduction of time-taking consumption implies substantial modification to the conventional decomposition of the gains from trade. This is because trade in produced goods can take place but not trade in time. While trade in goods expands agents’ financial affordability, their consumption time constraints remain. As a result, both the exchange and specialisation gains, as conventionally defined in the literature, vanish in the presence of a time-taking consumption. There are, however, positive gains from time allocation (shifting labour away from production toward consumption) and specialisation associated with that time reallocation.

For simplicity of analysis and ease of graphical illustration, many simplification assumptions have been made including the number of goods and countries, constant time rate of consumption time and representative agents. It should be apparent that the assumptions concerning the dimensionality of the model are not essential because the reasoning of the model remains valid when there are more than two goods or two countries. Similarly, assuming variable rates of consumption, the results of the model also continue to hold if the aggregate consumption-time budget curve is concave.

Heterogeneity of economic agents can be accommodated in a limited way as follows. If all agents have identical homothetic preferences and Leontief consumption technologies,
and only differ in labour or capital endowments, the analysis and propositions of the present model remain essentially unchanged. If agents have identical homothetic preferences and different consumption technologies but the ratio \( (a_2/a_1)^k \) remains constant for all agents \( k \), then the aggregate consumption-time budget continues to be linear so that the analyses in Sections 2 and 3 also remain largely valid.

References


